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ON ASSOCIATIVE SUPERALGEBRAS OF MATRICES

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1. Introduction. This work is a contribution to ongoing investigations of algebraic structures in relation to the theoretical description of physical systems. Matrix methods have been used by various mathematical physicists in the late nineteenth century and have been applied, for example, in the exploration of quaternions and other division algebras as a generalization of the complex number system for quantum physics, see, for example, [**25**]. One of the deepest results permeating physics is the spin-statistics theorem, see [**27**], according to which the space-time properties (spin) of elementary particles are correlated with their quantum statistical description. The two classes of particle statistics, Bose-Einstein and Fermi-Dirac, respectively, can be accommodated naturally in a larger algebraic scheme incorporating the notion of *grading* to accommodate various sign factors in defining relations (in this regard see, for example, [**3, 8, 15, 23, 24, 29, 30**]).

At the level of nonassociative algebras, the structure and representation theory of $\mathbf{Z}_2$-graded *Lie superalgebras* have been extensively studied as symmetry algebras of physical systems (for examples of applications we refer to [**2, 7, 11, 13, 14**]). In recent years the study of two-dimensional systems has led to the realization that richer algebraic schemes such as the so-called *quantum algebras* may be relevant (the spin-statistics theorem is also weaker in the two-dimensional case).

In the present paper we relax the notion of a superalgebra and investigate associative rings graded by semigroups. Retaining, in this paper, a bipartite decomposition of the underlying space into a 'Bose-like' and a 'Fermi-like' piece, we therefore study the five classes of two-element semigroups. Since matrix rings play important roles in this research direction (see [**1, 5, 10, 12, 26**]), the first natural step is to investigate the matrix algebras graded by the two-element semigroups.

Let S be a semigroup. An associative ring R is said to be S-*graded*,

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