ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 2, Summer 2004

WARPED PRODUCTS IN REAL SPACE FORMS

BANG-YEN CHEN

ABSTRACT. We prove a general inequality in terms of scalar curvature, the warping function, and the squared mean curvature for warped products isometrically immersed in real space forms. We also determine the warped products in real space forms which satisfy the equality case of the inequality.

1. Introduction. Let B and F be two Riemannian manifolds with Riemannian metrics g_B and g_F, respectively, and $f > 0$ be a differentiable function on B. Consider the product manifold $B \times F$ with its projection $\pi : B \times F \to B$ and $\eta : B \times F \to F$. The *warped product* $M = B \times_f F$ is the manifold $B \times F$ equipped with the Riemannian structure such that

$$\|X\|^2 = \|\pi_*(X)\|^2 + f^2(\pi(x))\|\eta * (X)\|^2 \tag{1.1}$$

for any tangent vector $X \in T_x M$. Thus we have $g = g_B + f^2 g_F$. The function f is called the *warping function* of the warped product (cf. [**15**]).

For a submanifold N in a Riemannian manifold $\widetilde{M}$ we denote by ∇ and $\widetilde{\nabla}$ the Levi-Civita connections of N and $\widetilde{M}$, respectively. The Gauss and Weingarten formulas are given respectively by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{1.2}$$

$$\widetilde{\nabla}_X \xi = -A\xi X + D_X \xi \tag{1.3}$$

for vector fields X, Y tangent to N and vector field ξ normal to N, where h denotes the second fundamental form, D the normal connection and A the shape operator of the submanifold.

Key words and phrases. Warped products, real space forms, inequality, rotation hypersurface, warping function, squared mean curvature.
2000 *Mathematics Subject Classification.* Primary 53C50, 53C42, Secondary 53B25.
Received by the editors on August 6, 2001, and in revised form on March 25, 2002.