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# THE RATIONALITY OF THE MODULI SPACES OF BIELLIPTIC CURVES OF GENUS FIVE WITH MORE BIELLIPTIC STRUCTURES

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**0. Introduction and notations.** Let $C$ be an irreducible, smooth, projective curve of genus $g \geq 2$, defined over the complex field $\mathbf{C}$. The curve $C$ is called *bielliptic* if it admits a degree 2 morphism $\pi: C \to E$ onto an elliptic curve $E$: such a morphism is called a *bielliptic structure*.

If $g \geq 6$ then the bielliptic structure is unique. If $g = 3, 4, 5$ this holds true generically, but there exist curves $C$ carrying more than one bielliptic structure.

We denote by $\mathfrak{M}_g^{be,n}$ the locus of points representing curves with at least $n$ bielliptic structures inside the coarse moduli space $\mathfrak{M}_g$ of smooth curves of genus $g$. There are the following sharp bounds: $n \leq 21, 10, 5$ if $g = 3, 4, 5$ respectively (see Corollary 5.8 of [**3**]).

We focus our interest on the case $g = 5$. It is already known that $\mathfrak{M}_5^{be,1}$ is rational (see [**6**]). The aim of this paper is to prove the following

**Main Theorem.** *The loci* $\mathfrak{M}_5^{be,2}$, $\mathfrak{M}_5^{be,3}$ *and* $\mathfrak{M}_5^{be,4} = \mathfrak{M}_5^{be,5}$ *are irreducible and rational of respective dimensions* 5, 4 *and* 2. $\square$

The loci $\mathfrak{M}_5^{be,n}$ play a helpful role in the description of the structure of the Chow ring $A(\mathfrak{M}_5)$ (see Section 4 of [**8**] where $\mathfrak{M}_5^{be,n} =: B_n$).

For the proof of the main theorem above we proceed imitating the method used in [**6**] for proving the rationality of $\mathfrak{M}_5^{be,1}$. Let $[C] \in \mathfrak{M}_5$ be the isomorphism class of a curve $C$. The canonical model $\widetilde{C}$ of $C$ is the base locus of a net of quadric hypersurfaces $\mathcal{N}$ in $\mathbf{P}^4_{\mathbf{C}}$. Let $N$ be a projective plane parametrizing the quadrics in $\mathcal{N}$. The discriminant curve $D \subseteq N$ of $\mathcal{N}$ is a stable plane quintic.

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