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MONADS AND BUNDLES ON RATIONAL SURFACES

NICHOLAS BUCHDAHL

ABSTRACT. A monad construction is presented for holomorphic bundles on an arbitrary blowup of $\mathbf{P}_2$ which have semi-stable direct image on $\mathbf{P}_2$. Three illustrative applications to different moduli problems are given.

1. Introduction. A *monad* M on a complex manifold X is a complex $0 \to A \xrightarrow{a} B \xrightarrow{b} C \to 0$ of holomorphic vector bundles with $a(x)$ injective and $b(x)$ surjective at each $x \in X$; the *cohomology* of M is the vector bundle $E(M) = \operatorname{Ker} b / \operatorname{Im} a$. The utility of monads lies in the fact that, under certain auspicious conditions, a vector bundle (or family of such) can be described as the cohomology of a monad (or family of such) of a particularly simple kind.

Horrocks [**13**] was the first to introduce monads and used them to show that every holomorphic vector bundle on $\mathbf{P}_n$ can be described by monads with A, B, C all projectively trivial, i.e., trivial twisted by a line bundle. Barth [**3**] used this to classify stable bundles on $\mathbf{P}_2$ up to linear algebraic data, and this work was extensively generalized and developed in the book [**15**]. The monad description of bundles on $\mathbf{P}_3$ was used by Atiyah et al. [**2**] in their celebrated description of instantons (self-dual solutions of the Yang-Mills equations) on S^4, using the Ward correspondence [**17**] between holomorphic bundles on $\mathbf{P}_3$ and the instantons on S^4. The close relationship between complex analytic geometry and gauge theory has provided a rich source for applications of monads, particularly in the context of computing moduli spaces.

Methods similar to those used for the ADHM construction were used in [**7**] to describe the instantons on $\mathbf{CP}_2$; in this case, instantons correspond to certain holomorphic vector bundles on the flag manifold

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