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NONDEGENERATE IDEALS IN FORMAL POWER SERIES RINGS

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ABSTRACT. We give a characterization of the ideals of finite colength of $\mathbf{C}[[x_1, \dots, x_n]]$ which have monomial integral closure using concepts from local algebra, thus leading to an algebraic proof of a result of Saia. We also construct a canonical minimal reduction of any ideal in the class of monomial ideals of $\mathbf{C}[[x_1, \dots, x_n]]$ of finite colength. Therefore, we give an effective method for the computation of the multiplicity of any ideal in this class.

1. Introduction. We denote by $\mathcal{A}$ the ring of formal power series $\mathbf{C}[[x_1, \dots, x_n]]$. By a result of Teissier [**15**], it is known that, when an ideal $I \subseteq \mathcal{A}$ of finite colength (that is, $\dim_{\mathbf{C}} \mathcal{A}/I < \infty$) is generated by monomials, then its multiplicity $e(I)$ is equal to $n! v(I)$, where $v(I)$ is the n-dimensional volume of the complementary in $\mathbf{R}^n_+$ of the Newton polyhedron of I. The equality $e(I) = n! v(I)$ is, in turn, equivalent to say that the integral closure of I is generated by those monomials x^k such that k belongs to the Newton polyhedron of I. In this work we characterize the ideals of $\mathbf{C}[[x_1, \dots, x_n]]$ satisfying this expression for the multiplicity. Then we give an algebraic proof of the main result of Saia in [**13**] that might be reproduced in more general contexts.

Motivated by the work of Yoshinaga [**17**] on the characterization of Newton nondegenerate functions, in the sense of Kouchnirenko [**7**], Saia established in [**13**] the definition of Newton nondegeneracy for ideals in the ring of convergent power series $\mathcal{O}_n$ and proved that, if $I \subseteq \mathcal{O}_n$ has finite colength, then I is Newton nondegenerate (or simply, *nondegenerate*, for short) if and only if the integral closure of I is determined by all the monomials with exponents in the Newton polyhedron of I. The proof that Saia gives of this result deals with the notion of toroidal embedding and the *growth condition* for the integral

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