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INFINITESIMAL NEIGHBORHOODS OF INFINITE-DIMENSIONAL COMPLEX PROJECTIVE SPACES

E. BALLICO

ABSTRACT. Let V be an infinite dimensional complex Banach space and Y a complex Banach manifold containing $X := \mathbf{P}(V)$ as a codimension r closed split submanifold. Assume that X admits smooth partitions of unity and that there is $\mathcal{O}_Y(1) \in \mathrm{Pic}\,(Y)$ such that $\mathcal{O}_Y(1)|X \cong \mathcal{O}_X(1)$. Fix an integer $n \geq 1$ and a finite rank holomorphic vector bundle E on the order n infinitesimal neighborhood $X^{(n)}$ of X in V. Set $s := \mathrm{rank}\,(E)$. Then $H^1(X^{(n)}, E) = 0$ and there are uniquely determined integers a_i, $1 \leq i \leq s$, such that $a_1 \geq \cdots \geq a_s$ and $E \cong \mathcal{O}_{X^{(n)}}(a_1) \oplus \cdots \oplus \mathcal{O}_{X^{(n)}}(a_s)$.

1. Introduction. Let V be a complex Banach space and $\mathbf{P}(V)$ the projective space of all its one-dimensional subspaces. We assume that $\mathbf{P}(V)$ admits smooth partitions of unity. For instance this is the case if V is a separable Hilbert space. Set $X := \mathbf{P}(V)$ and let Y be a complex Banach manifold containing X as a closed split submanifold; we recall that X is a split submanifold of Y if for every $P \in X$ there is an open neighborhood U of P in Y and a holomorphic submersion $f : U \to W$, W open neighborhood of 0 in $\mathbf{C}^r$ such that $U \cap X = f^{-1}(0)$. The integer r is the codimension of X in Y. For every integer $n \geq 0$ let $X^{(n)}$ be the infinitesimal neighborhood of order n of X in Y, i.e., the unreduced complex analytic subspace of Y with $\mathcal{I}_X^{n+1}$ as ideal sheaf; in the chart (U, f) with $f = (z_1, \dots, z_r)$ the complex space $U \cap X^{(n)}$ is defined by all monomials of degree $n+1$ in the variables $z_1, \dots, z_r$. We prove the following result.

Theorem 1. *Let V be an infinite dimensional complex Banach space such that $X := \mathbf{P}(V)$ admits smooth partitions of unity and Y a complex Banach manifold containing X as a codimension r closed*

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