

SPACE-LIKE EINSTEIN KÄHLER SUBMANIFOLDS IN AN INDEFINITE COMPLEX HYPERBOLIC SPACE

YONG-SOO PYO

ABSTRACT. The purpose of this paper is to study space-like Einstein Kähler submanifolds with restricted full immersions and parallel second fundamental forms in an indefinite complex hyperbolic space.

1. Introduction. The theory of semi-definite complex submanifolds of a semi-definite complex space form is one of the most interesting research subjects in differential geometry and it is studied by many geometers from the various points of view, see [1–3, 10–12] and [14], for instance.

As one of such studies, in their paper [10], Nakagawa and Takagi classified completely locally symmetric Kähler submanifolds of a complex projective space. In particular, it is seen that complex submanifolds whose second fundamental form are parallel of a complex projective space are all Einstein. Conversely, Einstein Kähler submanifolds of a complex space form do not satisfy necessarily the result that the second fundamental form is parallel, and it is seen in [10] that there exist many Einstein Kähler submanifolds of a complex projective space whose second fundamental form are not necessarily parallel. Furthermore, Romero [13] and Umehara [15] independently proved the indefinite version and they found that there exists a full holomorphic isomorphic immersion of an indefinite complex space form $M_s^n(c)$ into an indefinite complex space form $M_{s+t}^{n+p}(c')$.

On the other hand, Einstein Kähler submanifolds of a complex projective space whose second fundamental form are parallel were investigated by Nakagawa [9]. He proved the following

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