COMPACT MULTIPLICATION OPERATORS ON WEIGHTED SPACES OF VECTOR-VALUED CONTINUOUS FUNCTIONS

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ABSTRACT. In this note we characterize the compact multiplication operators M_π on the weighted locally convex spaces $CV_0(X, E)$ of vector-valued continuous functions induced by the operator-valued mappings $\pi : X \to B(E)$.

0. Introduction. Let $L(X, E)$ be a vector space of functions from a nonempty set X to a vector space E over the field $\mathbf{C}$ or $\mathbf{R}$. Let $T(E)$ be a set of linear transformations from E to itself. If $\phi : X \to X$ and $\pi : X \to T(E)$ are mappings such that the weighted composite function $\pi.f \circ \phi$ belongs to $L(X, E)$, whenever $f \in L(X, E)$, then the mapping taking f to $\pi.f \circ \phi$ is a linear transformation on $L(X, E)$ and we denote it by $W_{\pi,\phi}$. In case $L(X, E)$ is a topological vector space and the mapping $W_{\pi,\phi}$ is continuous, we call $W_{\pi,\phi}$ the weighted composition operator on $L(X, E)$ induced by the symbol (π, ϕ). In case ϕ is the identity map, we call it the multiplication operator induced by π and we denote it by M_π. For details on these operators we refer to [**11**].

The compact weighted composition operators on spaces of continuous functions have been studied extensively by many authors like Kamowitz [**5**], Feldman [**3**], Singh and Summers [**13**], Jamison and Rajagopalan [**4**], Takagi [**14**], Chan [**2**] and Singh and Manhas [**12**]. As we know, the class of weighted composition operators include the class of multiplication operators and the class of composition operators. One natural question arises: is it possible to get the behavior of the compact multiplication operators from the study of the compact weighted composition operators? In general, it is not possible since the conditions obtained earlier for a weighted composition operator to be compact is not satisfied by the identity map ϕ. So it motivates us to