ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 3, Fall 2004

RELATIONS FOR IMAGINARY PARTS OF ZEROS OF ENTIRE FUNCTIONS

M.I. GIL'

ABSTRACT. Finite order entire functions are considered. New relations for the imaginary parts of the zeros are derived. They particularly generalize the Cartwright-Levinson theorem. By virtue of these relations, under some restriction, the Hadamard theorem on the convergence exponent of the zeros is improved.

1. The main result. Consider the finite order entire function

$$f(\lambda) = \sum_{k=0}^{\infty} \frac{a_k \lambda^k}{(k!)^\gamma}, \quad \lambda \in \mathbf{C}, \quad a_0 = 1, \quad \gamma > 0, \tag{1.1}$$

with complex, in general, coefficients. Assume that

$$w(f) \equiv \sum_{k=1}^{\infty} |a_k|^2 < \infty \tag{1.2}$$

and put

$$\psi_f \equiv \left[|Im\ a_1|^2 + \sum_{k=2}^{\infty} |a_k|^2 \right]^{1/2}.$$

Everywhere below $\{z_k(f)\}_{k=1}^m$, $m \le \infty$, is the set of all the zeros of f taken with their multiplicities. In this section it is assumed that the zeros are numerated in the following way

$$\left| Im \frac{1}{z_k(f)} \right| \ge \left| Im \frac{1}{z_{k+1}(f)} \right|, \quad k = 1, \dots, m-1.$$

In the sequel, if $m < \infty$, then we take $|z_k(f)|^{-1} = 0$ for $k > m$.

AMS *Mathematics Subject Classification.* 30D20, 30C15.
Key words and phrases. Entire functions, estimates for zeros.
This research was supported by the Israel Ministry of Science and Technology.
Received by the editors on November 18, 1999.