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POLYNOMIAL CHARACTERIZATION OF THE COMPACT RANGE PROPERTY

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ABSTRACT. Among other results it is proved that, for a Banach space F and an integer m, the following assertions are equivalent:

(a) F has the compact range property;

(b) for every Banach space E, each m-homogeneous Pietsch integral polynomial from E into F is compact;

(c) every m-homogeneous 1-dominated polynomial from $C([0,1])$ into F is compact;

(d) every m-homogeneous polynomial from $L_1([0,1])$ into F is completely continuous.

A Banach space F is said to have the *compact range property* (CRP, for short) if every F-valued countably additive measure of bounded variation has compact range [**15**]. Every Banach space with the weak Radon-Nikodým property has the CRP. A dual Banach space has the CRP if and only if its predual contains no copy of l_1. We refer to [**9, 10, 15, 17**] for more about the CRP.

We recall the following characterizations of the CRP in terms of (linear bounded) operators:

Theorem 1. *For a Banach space F the following facts are equivalent*:

(a) *F has the* CRP;

(b) *for any compact Hausdorff space K, every absolutely summing operator from $C(K)$ into F is compact*;

(c) *every absolutely summing operator from $C([0,1])$ into F is compact*;

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