ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 3, Fall 2004

SPACES OF OPERATORS, c_0 AND l^1

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ABSTRACT. If Y is a Banach space so that l^1 embeds isomorphically as a complemented subspace of the separable space Y^* but c_0 does not embed as a subspace of Y, then it is shown that there is an infinite dimensional Banach space X so that l^1 embeds complementably in $X \otimes_\gamma Y^*$ but c_0 does not embed in $L(X, Y)$.

In a classic paper on the structure of Banach spaces [**2**], Bessaga and Pelczynski established the following result.

Theorem 1. *If c_0 embeds isomorphically in the dual X^* of the Banach space X, then l^∞ embeds in X^* and l^1 embeds complementably in X.*

The following complete generalization of Theorem 1 was established in [**7**]. In this theorem (e_n^*) denotes the canonical unit vector basis of l^1 and $X \otimes_\gamma Y^*$ denotes the greatest crossnorm tensor product completion of X and Y^*.

Theorem 2. *If X is an infinite dimension Banach space and c_0 embeds in $L(X, Y)$, then l^∞ embeds in $L(X, Y)$ and there is an isomorphism $J : l^1 \to X \otimes_\gamma Y^*$ so that $J(l^1)$ is complemented in $X \otimes Y^*$ and $J(e_n^*)$ is a finite rank tensor for each n.*

Of course, the converse of Theorem 1 is immediate, i.e., if l^1 embeds complementably in X, then certainly l^∞ (and thus c_0) embeds in X^*. The status of the converse of Theorem 2 is not clear at all. There is an example on page 215 of [**7**] which purports to show that the complementability of l^1 in the greatest crossnorm tensor product completion of X and Y^* does not imply that c_0 embeds in the space $L(X, Y)$ of all bounded linear transformations from X to Y. However,

AMS *Mathematics Subject Classification.* 46B20, 46B25.
Received by the editors on December 14, 2001.