

INTERVAL OSCILLATION CRITERIA FOR SECOND ORDER NONLINEAR DELAY DIFFERENTIAL EQUATIONS

QIGUI YANG AND RONALD M. MATHSEN

ABSTRACT. New oscillation criteria established in this paper for the second order nonlinear equations

$$(r(t)\psi(x(t))x'(t))' + F(t, x(t), x'(t), x(\tau(t)), x'(\tau(t))) = 0$$

are different from most known ones in the sense that they are based on the information only on a sequence of subintervals of $[t_0, \infty)$ rather than on the whole half-line. Our results are more general and sharper than some previous results and handle the cases which are not covered by known results. Several examples that show the generality of our results are also included.

1. Introduction. We are concerned here with the oscillatory behavior of solutions of the second order nonlinear differential equation (1)

$$(r(t)\psi(x(t))x'(t))' + F(t, x(t), x'(t), x(\tau(t)), x'(\tau(t))) = 0, \quad t \geq t_0$$

where $F : [t_0, \infty) \times R^4 \rightarrow R$ is a continuous function. In what follows, we always assume without mention that

- (A₁) $r : I = [t_0, \infty) \rightarrow (0, \infty)$ is continuously differentiable;
- (A₂) $\psi : R \rightarrow R$ is continuously differentiable and $\psi(x) > 0$ for $x \neq 0$;
- (A₃) $\tau : I \rightarrow R$ is continuously differentiable with $\tau'(t) > 0$ for all $t \in I$, $\tau(t) \leq t$ for $t \geq t_0$ and $\lim_{t \rightarrow \infty} \tau(t) = \infty$;
- (A₄) there exist functions q, f_0, f and g such that

$$\begin{aligned} &F(t, x(t), x'(t), x(\tau(t)), x'(\tau(t))) \operatorname{sgn} x \\ &\geq q(t)f_0(x(t))f(x(\tau(t)))g(x'(t), x'(\tau(t))) \operatorname{sgn} x \end{aligned}$$

2000 AMS *Mathematics Subject Classification.* 34C10, 34C15.

Key words and phrases. Second order, nonlinear delay differential equations, oscillation.

Received by the editors on June 30, 2002, and in revised form on September 8, 2002.