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OSCILLATION CRITERIA OF KNESER-HILLE TYPE FOR SECOND-ORDER DIFFERENTIAL EQUATIONS WITH NONLINEAR PERTURBED TERMS

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ABSTRACT. This paper is concerned with the oscillation problem for nonlinear differential equations of Euler type, which are denoted by (E_n) with $n = 1, 2, 3, \ldots$. Equation (E_n) consists of a linear main term and a nonlinear perturbed term. If the nonlinear perturbation vanishes, then all nontrivial solutions of (E_n) are nonoscillatory. A pair of sufficient and necessary conditions on the perturbed term for all nonlinear solutions of (E_n) to be oscillatory is given. It is also proved that all solutions of (E_n) tend to zero.

1. Introduction. The existence and number of the zeros of the solutions of ordinary differential equations are an important subject in the qualitative theory. By an *oscillatory solution* we mean one having an infinite number of zeros on $0 \le t < \infty$. Otherwise, the solution is called *nonoscillatory*.

For example, we consider the Euler differential equation with positive damping

$$y'' + \frac{2}{t}\,y' + \frac{\delta}{t^2}\,y = 0, \tag{L_1}$$

where $' = d/dt$. Then we see that all nontrivial solutions of (L_1) are nonoscillatory if and only if $\delta \le 1/4$. In fact, equation (L_1) has the general solution

$$y(t) = \begin{cases} \dfrac{1}{\sqrt{t}}\,(K_1 t^{\zeta} + K_2 t^{-\zeta}) & \text{if } \delta \ne \dfrac{1}{4}, \\ \dfrac{1}{\sqrt{t}}\,(K_3 + K_4 \log t) & \text{if } \delta = \dfrac{1}{4}, \end{cases}$$

where K_i, $i = 1, 2, 3, 4$, are arbitrary constants and ζ is a number satisfying

$$\frac{1}{4} - \zeta^2 = \delta. \tag{1.1}$$

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