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A FIX-FINITE APPROXIMATION THEOREM

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ABSTRACT. We prove that if C_i is a nonempty convex compact subset of a metrizable locally convex vector space for $i = 1, \dots, m$ such that $\cap_{i=1}^{i=m} C_i \neq \varnothing$ or $C_i \cap C_j = \varnothing$ for $i \neq j$, then for every $\varepsilon > 0$ and for every n-valued continuous multi-function $F : \cup_{i=1}^{i=m} C_i \to \cup_{i=1}^{i=m} C_i$ there exists an n-valued continuous multi-function $G : \cup_{i=1}^{i=m} C_i \to \cup_{i=1}^{i=m} C_i$ which is ε-near to F and has only a finite number of fixed points.

1. Introduction and preliminaries. Let A be a nonempty subset of a metrizable locally convex vector space. We say that A satisfies the fix-finite approximation property, FFAP, for a family $\mathcal{F}$ of self multi-functions, or self-maps, of A if, for every $F \in \mathcal{F}$ and all $\varepsilon > 0$, there exists $G \in \mathcal{F}$ which is ε-near to F and has only a finite number of fixed points.

In [**3, 5**], Hopf proved by a special construction that any finite polyhedron which is connected and has dimension greater than one satisfies the FFAP for the family of continuous self-maps. In [**8**], Schirmer extended this result to any n-valued continuous self multi-function. In [**1**], Baillon and Rallis showed that any finite union of closed convex subsets of a Banach space satisfies the FFAP for the family of compact self-maps.

In this paper we consider the more general case of metrizable locally convex vector spaces. Our first key result, Theorem 2.2, is a generalization of a theorem of Baillon-Rallis [**1**]. The main result in this paper is Theorem 3.7: Let C_i be a nonempty convex compact subset of a metrizable locally convex vector space for $i = 1, \dots, m$ such that $\cap_{i=1}^{i=m} C_i \neq \varnothing$ or $C_i \cap C_j = \varnothing$ for $i \neq j$; then $\cup_{i=1}^{i=m} C_i$ satisfies the FFAP for any n-valued continuous multi-function $F : \cup_{i=1}^{i=m} C_i \to \cup_{i=1}^{i=m} C_i$.

In the sequel we recall some definitions and well-known results for subsequent use. Let $\varepsilon > 0$ and let X be a topological space and (Y, d)

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