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EXPONENTIAL FUNCTION ANALOGUE OF KLOOSTERMAN SUMS

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ABSTRACT. We consider exponential sums of the form

$$\mathcal{K}_g(a,b) = \sum_{\substack{x=1 \\ \gcd(x,t)=1}}^{t} \exp\left(2\pi i(ag^x + bg^{x^{-1}})/p\right),$$

where g is of multiplicative order t modulo the prime p. We obtain a nontrivial upper bound on these sums on average over all elements g of multiplicative order t, provided that $t \geq p^{3/4+\delta}$ with an arbitrary fixed $\delta > 0$.

1. Introduction. Let p be a prime, and let $\mathbf{F}_p$ be a finite field of p elements. For an integer $t \geq 1$ we denote by $\mathbf{Z}_t = \{0, \dots, t-1\}$ the residue ring modulo t and we denote by $\mathbf{Z}_t^*$ the subset of $\mathbf{Z}_t$ consisting of $\varphi(t)$ invertible elements, where $\varphi(t)$ is the Euler function. We also identity $\mathbf{F}_p$ with the set $\{0, \dots, p-1\}$.

Finally we define $\mathbf{e}(z) = \exp(2\pi i z/p)$ and use $\log z$ for the natural logarithm of z.

For a divisor $t|p-1$ we denote by $\mathcal{U}_t$ the set of elements $g \in \mathbf{F}_p^*$ of multiplicative order t, that is,

$$\mathcal{U}_t = \{g \in \mathbf{F}_p^* \mid g^s \neq 1,\ 1 \leq s < t;\ g^t = 1\}.$$

It is easy to see that $\#\mathcal{U}_t = \varphi(t)$.

For $g \in \mathcal{U}_t$, we consider exponential sums

$$\mathcal{K}_g(a,b) = \sum_{x \in \mathbf{Z}_t^*} \mathbf{e}(ag^x + bg^{x^{-1}}),$$

where $a, b \in \mathbf{F}_p$.

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