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ON TRANSFORMATION LAWS FOR THETA FUNCTIONS

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ABSTRACT. We determine transformation laws for theta functions of higher degree.

1. Introduction. Siegel [**8–10**], for example, proves transformation laws for theta functions (depending on a single complex variable) attached to quadratic forms. If the quadratic form is indefinite, then Siegel's definition of the theta function also depends on a majorant of the quadratic form, an idea that Siegel credits to Hermite. Siegel's results have been generalized and transformation laws for theta functions of higher degree have been established. Andrianov and Maloletkin [**1**] and [**2**] use Eichler's "embedding trick" to determine transformation properties of theta series, depending on one complex $n \times n$ matrix variable, corresponding to positive definite and also indefinite quadratic forms: they use Eichler's method, see [**5**], for example, of recognizing such theta series as specializations of symplectic theta functions. Ziegler [**14**] develops a theory of holomorphic Jacobi forms of higher degree and shows that theta functions (depending on two complex matrix variables) attached to positive definite quadratic forms are examples of such forms.

The purpose of this paper is to show that Eichler's "embedding trick" can also be applied to generalize Ziegler's result. We will define $\Theta_{F,H,\zeta}(Z,W)$, a theta function of higher degree, depending on a complex $n \times n$ matrix variable Z and a complex $j \times n$ matrix variable W, attached to an indefinite quadratic form, and we will determine the behavior of $\Theta_{F,H,\zeta}(Z,W)$ under modular transformations by proceeding as in Andrianov and Maloletkin [**1**] and [**2**], see also [**7**]. Friedberg [**6**] defines a modified version of the usual symplectic theta function, $\vartheta(Z, \left(\begin{smallmatrix} u \\ v \end{smallmatrix}\right), w, f)$, and he proves a transformation formula for his function. We state that transformation formula in a slightly more general way and show that certain coefficients of $\Theta_{F,H,\zeta}(Z,W)$ can be regarded

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