ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 4, Winter 2004

# EXTREME POINTS AND THE DIAMETER NORM

JUAN J. FONT AND MANUEL SANCHIS

ABSTRACT. Let $A$ be a point-separating closed linear subspace of $C(X)$, $X$ compact, which contains the constants. We characterize the extreme points of the closed unit ball of $A$ when endowed with the diameter (semi) norm.

**1. Introduction.** Let $X$ be a (infinite) compact Hausdorff space. As usual, $C(X)$ stands for the space of all continuous real-valued functions on $X$ endowed with the supremum norm $\|\cdot\|_\infty$. The diameter (semi) norm of any function $f$ in $C(X)$ is defined to be the diameter of the range of $f$. This norm was first studied by Győry and Molnár in [**9**] when dealing with linear bijections of $C(X)$ ($X$ compact and first countable) which leave the diameter of the range of every function invariant, which is to say, diameter preserving mappings. Since then, considerable attention has been given to these mappings and to the diameter norm. Namely, González and Uspenskij ([**7**]) and, independently, Cabello ([**2**]) removed the hypothesis of first countability on $X$ and characterized the extreme points of the closed unit ball of the dual of $C(X)$ endowed with the diameter norm (see also [**6**] for similar results on certain subspaces of continuous functions). Rao and Roy [**10**] have obtained analogue characterizations in the context of spaces of affine functions and vector-valued continuous functions. Recently, in [**3**], the authors have proved that the diameter norm is *maximal* on $C_0(X)$, where $X$ is a connected non-compact manifold.

Let $A$ be a point-separating closed linear subspace of $(C(X), \|\cdot\|_\infty)$ which contains the constant functions. Let $\mathcal{C}$ denote the constant functions on $X$. In this paper we study the extreme points of the closed unit ball of the dual of the quotient $A/\mathcal{C}$ endowed with the diameter norm.

2000 AMS *Mathematics Subject Classification.* 46A55, 46E15.
*Key words and phrases.* Extreme point, diameter norm.
Research partially supported by Spanish Ministry of Science and Technology (BFM2000-0913), Generalitat Valenciana (GV00-063-1) and Bancaixa (P1-1B2001-08).
Received by the editors on January 15, 2002, and in revised form on May 20, 2002.