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SELECTION OF SLOW DIFFUSION IN A REACTION DIFFUSION MODEL: LIMITING CASES

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Introduction. A common phenomenon, observed in a wide variety of models for the dispersal of organisms, is that dispersal rates tend to be lower if the environment is spatially heterogeneous [**10**]. In an attempt to understand this, Dockery et al. [**3**] proposed a reaction diffusion model for the evolution of n different phenotypes of a species, where the only phenotypic difference is in the diffusion rate. It is assumed that the per-capita net rate of increase of each phenotype, denoted by a, is not a constant. The diffusion rates are $d_1 < d_2 < \cdots < d_n$, the environment is a region $\Omega \subset \mathbf{R}^k$. It is assumed that Ω is a bounded domain with smooth boundary, across which there is no migration. In this model, the equation for the density $u_i(x,t)$ of phenotype i is:

$$\frac{\partial u_i}{\partial t} = d_i \Delta u_i + \left[a(x) - \sum_{j=1}^{n} u_j\right] u_i \quad \text{in } \Omega, \quad i = 1, \dots, n. \tag{1}$$

with homogeneous Neumann boundary condition: $\partial u_i / \partial \nu = 0$ and prescribed initial values $u_i(x,0)$ which are nonnegative functions in Ω.

One of the basic results of [**3**] is that the only nonnegative equilibria of this system are semi-trivial solutions, i.e., they have the form $\tilde{U}^i(x)$, where the jth component is zero for $j \neq i$ and the ith component $\tilde{U}^i_i(x)$ is the positive solution of

$$\begin{aligned} d_i \Delta u + [a(x) - u]\, u &= 0 \quad \text{in } \Omega \\ \frac{\partial u}{\partial \nu} &= 0 \quad \text{on } \partial\Omega. \end{aligned} \tag{2}$$

It was also shown that $\tilde{U}^1(x)$ is linearly asymptotically stable and $\tilde{U}^i(x)$ is unstable for $i = 2, \dots, n$. Furthermore, if $n = 2$ and the

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