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INEQUALITIES OF OSTROWSKI TYPE IN TWO DIMENSIONS

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ABSTRACT. A weighted version of Ostrowski type inequality in two dimensions is established. An ordinary generalization of Ostrowski's inequality in two dimensions and a corresponding Ostrowski-Grüss inequality are also derived.

1. Introduction. In 1938 A. Ostrowski proved the following integral inequality, [**15**] or [**14**, p. 468].

Theorem 1. *Let $f : I \to R$, where $I \subset R$ is an interval, be a mapping differentiable in the interior $Int\ I$ of I, and let $a, b \in Int\ I$, $a < b$. If $|f'(t)| \le M$, for all $t \in [a, b]$ then we have*

$$(1) \qquad \left| f(x) - \frac{1}{b-a}\int_a^b f(t)\,dt \right| \le \left[\frac{1}{4} + \frac{(x-(a+b/2))^2}{(b-a)^2} \right](b-a)M,$$

for $x \in [a, b]$.

The first (direct) generalization of Ostrowski's inequality was given by Milovanović and Pečarić in [**12**]. In recent years a number of authors have written about generalizations of Ostrowski's inequality. For example, this topic is considered in [**1, 3, 5, 7**] and [**12**]. In this way some new types of inequalities are formed, such as inequalities of Ostrowski-Grüss type, inequalities of Ostrowski-Chebyshev type, etc. The first inequality of Ostrowski-Grüss type was given by Dragomir and Wang in [**5**]. It was generalized and improved in [**7**]. Cheng gave a sharp version of the mentioned inequality in [**3**]. The first multivariate version of Ostrowski's inequality was given by Milovanović in [**10**], see also [**11**] and [**14**, p. 468]. Multivariate versions of Ostrowski's

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