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LIÉNARD LIMIT CYCLES ENCLOSING PERIOD ANNULI, OR ENCLOSED BY PERIOD ANNULI

M. SABATINI

ABSTRACT. We construct examples of polynomial Liénard systems with both centers and limit cycles. The first class of examples has limit cycles enclosed by period annuli. The second class has limit cycles surrounding central regions. In both cases we show that it is possible to construct polynomial systems having an arbitrary number of limit cycles with such properties. As a limit case, we construct an analytic Liénard system with infinitely many limit cycles surrounding a central region. We also show that for every n there exists a Liénard system of degree n with $n-2$ limit cycles.

1. Introduction. Let

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y) \tag{1}$$

be an autonomous plane differential system. We assume $P(x,y)$, $Q(x,y)$ to be analytic real functions defined on all of the real plane. We say that a critical point O of (1) is a center if it has a punctured neighborhood covered with nontrivial cycles. If O is a center, the largest connected region covered with cycles surrounding O is called *central region* and will be denoted by N_O. Every connected region covered with nontrivial concentric cycles is usually called a *period annulus*. Period annuli are not necessarily contained in central regions. An example is given by the Hamiltonian system

$$\dot{x} = y, \qquad \dot{y} = -x(4x^2-1)(x^2-1), \tag{2}$$

which has centers at $(-1,0)$, $(0,0)$, $(1,0)$, and a period annulus enclosing such centers. Since when O is a center there exists a first integral

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