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DISTRIBUTION OF MINIMAL VARIETIES IN SPHERES IN TERMS OF THE COORDINATE FUNCTIONS

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ABSTRACT. Let M be a compact k-dimensional Riemannian manifold minimally immersed in the unit n-dimensional sphere S^n. It is easy to show that for any $p \in S^n$ the boundary of the geodesic ball in S^n with radius $\pi/2$ and center at p (in this case this boundary is an equator) must intercept the manifold M. When the codimension is 1, i.e., $k = n-1$, it is known that the Ricci curvature is not greater than 1. We will prove that if the Ricci curvature is not greater than $1-\alpha^2/(n-2)$, then the boundary of every geodesic ball with radius $\cot^{-1}(\alpha)$ must intercept the manifold M. We give examples of manifolds for which the radius $\cot^{-1}(\alpha)$ is optimal. Next, for any codimension, i.e., for any $M^k \subset S^n$, we find a number r_1 that depends only on n such that for any collection of $n+1$ points $\{p_i\}_{i=1}^{n+1}$ in S^n that constitutes an orthonormal basis of $\mathbf{R}^{n+1}$, the union of the boundaries of the geodesic balls with radius r_1 and center p_i, $i = 1, 2, \dots, n+1$, must intercept the manifold M.

1. Introduction and preliminaries. Let M be a compact, oriented minimal hypersurface immersed in the n-dimensional unit sphere S^n. Let ν be a unit normal vector field along M. For any tangent vector $v \in T_mM$, $m \in M$, the shape operator A is given by $A(v) = -\overline{D}_v\nu$ where $\overline{D}$ denotes the Levi Civita connection in $\mathbf{R}^{n+1}$. With the same notation, for any tangent vector field W, the Levi Civita connection on M is given by $D_vW = (\overline{D}_vW)^T$ where $(\quad)^T$ denotes the orthogonal projection from $\mathbf{R}^{n+1}$ to T_mM. For a function $f : M \to \mathbf{R}$, ∇f will denote the gradient of f. For any pair of vectors $v, w \in T_mM$ the Hessian of f is given by $H(f)(v,w) = \langle D_v\nabla f, w\rangle$, where $\langle\quad,\quad\rangle$ denotes the inner product in $\mathbf{R}^{n+1}$. The Laplacian of f is given by $\Delta(f) = \sum_{i=1}^{n-1} H(f)(v_i, v_i)$ where $\{v_i\}_{i=1}^{n-1}$ is an orthonormal basis of T_mM.

For a given $w \in \mathbf{R}^{n+1}$, let us define the functions $l_w : M \to \mathbf{R}$ and $f_w : M \to \mathbf{R}$ by $l_w(m) = \langle m, w\rangle$ and $f_w(m) = \langle \nu(m), w\rangle$. Clearly

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