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ON POINT VALUES OF BOEHMIANS

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ABSTRACT. The notion of a value of a Boehmian at a point, its properties and the concept of regular delta sequences are available in the literature. Let $\mathcal{E}$ be a Banach space. Denote by $C(\mathbf{R}^N, \mathcal{E})$ the space of all continuous $\mathcal{E}$-valued functions on $\mathbf{R}^N$ and by $\mathcal{D}(\mathbf{R}^N)$ the space of all infinitely differentiable real-valued functions with compact support in $\mathbf{R}^N$. Using $C(\mathbf{R}^N, \mathcal{E})$ as the top space and the usual delta sequences from $\mathcal{D}(\mathbf{R}^N)$ we can construct in a canonical way a Boehmian space $\mathcal{B} = \mathcal{B}(\mathbf{R}^N, \mathcal{E})$. In 1994, Piotr Mikusiński and Mourad Tighiouart asserted that, if for every representation $[f_n/\phi_n]$ of $F \in \mathcal{B}$ where (ϕ_n) is regular delta sequence we have $\lim_{n\to\infty} f_n(x_0) = a$, then $F(x_0) = a$. In this paper we shall point out that the proof of this theorem contains an error, produce a counterexample to show that the theorem is not valid and obtain modified conditions for its validity. As a consequence we shall also show that if $F = [f_n/\phi_n]$ where (ϕ_n) is a delta sequence made of one function and if $\lim_{n\to\infty} f_n(x_0) = a$ for every such representation, then F need not have a value at x_0. Incidentally, this observation settles one of the questions raised by Piotr Mikusiński and Mourad Tighiouart.

1. Introduction. The concept of Boehmians was first introduced and studied in [**3**]. Various spaces of Boehmians and their properties were available in the literature, for example, see [**2**]. In [**4**] the notion of a value of a Boehmian at a point was defined and its properties were studied. Further related results can be found in [**1**]. An equivalent condition for a Boehmian to have a value at a point is claimed in Theorem 2.4 of [**4**]. We shall first point out that the proof of this theorem contains an error. In addition we shall produce a counterexample and establish that this theorem is not valid. We shall also explain how the hypothesis of this theorem must be modified for

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