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# BOUNDS FOR THE FABER COEFFICIENTS OF CERTAIN CLASSES OF FUNCTIONS ANALYTIC IN AN ELLIPSE

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ABSTRACT. Let $\Omega$ be a bounded, simply connected domain in $\mathbf{C}$ with $0 \in \Omega$ and $\partial\Omega$ analytic. Let $S(\Omega)$ denote the class of functions $F(z)$ which are analytic and univalent in $\Omega$ with $F(0) = 0$ and $F'(0) = 1$. Let $\{\Phi_n(z)\}_{n=0}^{\infty}$ be the Faber polynomials associated with $\Omega$. If $F(z) \in S(\Omega)$, then $F(z)$ can be expanded in a series of the form

$$F(z) = \sum_{n=0}^{\infty} A_n \Phi_n(z), \quad z \in \Omega$$

in terms of the Faber polynomials. Let

$$E_r = \left\{(x, y) \in \mathbf{R}^2 : \frac{x^2}{(1 + (1/r^2))^2} + \frac{y^2}{(1 - (1/r^2))^2} < 1\right\},$$

where $r > 1$. In this paper we obtain sharp bounds for the Faber coefficients $A_0$, $A_1$ and $A_2$ of functions $F(z)$ in $S(E_r)$ and in certain related classes.

**1. Introduction.** Let $S$ denote the class of functions $f$ analytic and univalent in the unit disk $\mathbf{D} = \{z : |z| < 1\}$ such that

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

The Bieberbach conjecture [**2**] asserts that if $f \in S$, then $|a_n| \leq n$, $n \geq 2$. This famous conjecture was proved by de Branges [**3**] in 1984.

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