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MULTILINEAR TRIF d-MAPPINGS IN BANACH MODULES OVER A C^*-ALGEBRA

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ABSTRACT. We define a multilinear Trif d-mapping, and prove the stability of multilinear Trif d-functional equations in Banach modules over a unital C^*-algebra.

1. Introduction. Let E_1 and E_2 be Banach spaces with norms $\|\cdot\|$ and $\|\cdot\|$, respectively. Consider $f : E_1 \to E_2$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbf{R}$ for each fixed $x \in E_1$. Assume that there exist constants $\varepsilon \geq 0$ and $p \in [0,1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon(||x||^p + ||y||^p)$$

for all $x, y \in E_1$. Rassias [**4**] showed that there exists a unique $\mathbf{R}$-linear mapping $T : E_1 \to E_2$ such that

$$\|f(x) - T(x)\| \leq \frac{2\varepsilon}{2-2^p}\, ||x||^p$$

for all $x \in E_1$.

Recently, Trif [**6**, Theorem 2.1] proved that, for vector spaces V and W, a mapping $f : V \to W$ with $f(0) = 0$ satisfies the functional equation

$$\text{(A)} \quad n\, {}_{n-2}C_{k-2} f\left(\frac{x_1 + \cdots + x_n}{n}\right) + {}_{n-2}C_{k-1} \sum_{l=1}^{n} f(x_l) = k \sum_{1 \leq l_1 < \cdots < l_k \leq n} f\left(\frac{x_{l_1} + \cdots + x_{l_k}}{k}\right)$$

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