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NIMLIKE GAMES WITH GENERALIZED BASES

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ABSTRACT. In several earlier papers [**4**] and [**5**], we discussed single pile games of Nim in which the number of counters that can be removed varies during the play of the game. In [**6**] we showed how to effectively play the single-pile game in which the number of counters that can be removed is a function of the number removed on the previous move. In that paper we constructed a number base and showed that in the winning strategy, the winning player can reduce the number of summands in a certain representation of the current pile size. In this paper, we reverse the situation by starting with an arbitrary base, and then construct a game whose winning positions are determined by the base. In particular, the winning strategy for such games consists of reducing the number of summands in the representation, with respect to this base, of the current pile size. In the Appendix we have worked through an example that illustrates all of the concepts given in this paper.

1. Definition. A *number base* is a strictly increasing sequence $B = (b_0 = 1, b_1, b_2, \dots)$ of positive integers. B can be finite or infinite.

In this paper we will consider B to be infinite. The reader can prove analogous results when B is finite. The following theorem is well known. The proof is given for the sake of completeness.

Theorem. *Let B be an infinite number base. Then each positive integer N can be represented as $N = b_{i_1} + b_{i_2} + \cdots + b_{i_t}$, where $b_{i_1} \le b_{i_2} \le \cdots \le b_{i_t}$ and each b_{i_j} belongs to B, by the following recursive algorithm.*

First, we represent the number 1 by $1 = b_0$. If $1, 2, 3, \dots m-1$ have been represented by the algorithm, then m can be represented as follows: Let b_k denote the largest element of B not exceeding m. That is, $b_k \le m < b_{k+1}$. Then $m = (m - b_k) + b_k$ and $m - b_k < b_{k+1}$. If $m - b_k = 0$ then the algorithm is finished.

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