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THE TRANSCENDENCE OF CERTAIN INFINITE SERIES

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ABSTRACT. The paper deals with two criteria concerning the transcendence of the sums of infinite series. The terms of these series consist of positive rational numbers which converge rapidly to zero. Some examples are provided which make use of sequences defined recursively.

1. Introduction. There are several methods one may use to prove the transcendence of infinite series. One of them is Mahler's method. A nice survey of such results can be found in Nishioka's book [**5**]. Another technique makes use of the Roth's theorem [**8**]. One can also use Nyblom's theorem which can be found in [**6**]. Later he proved in [**7**] that if $\lambda > 2$ is a fixed real number and $\{a_k\}_{k=1}^{\infty}$ is a sequence of integers greater than unity and such that

$$\liminf_{k\to\infty} \frac{a_{k+1}}{a_k^{\lambda+1}} > 1 \tag{1}$$

then the series $\sum_{k=1}^{\infty} 1/a_k$ converges to a transcendental number. If we want to describe a general criterion for the transcendence of series which converge quickly then it is useful to introduce the concept of transcendental sequences.

Definition 1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers. If for every sequence $\{c_n\}_{n=1}^{\infty}$ of positive integers the number $\sum_{n=1}^{\infty} 1/(a_n c_n)$ is transcendental, then the sequence $\{a_n\}_{n=1}^{\infty}$ is called transcendental.

This definition is due to Hančl [**3**]. Some criteria for transcendental sequences can be found in the same paper or in [**4**]. One interesting

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