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COUNTING GENERALIZED ORDERS ON NOT NECESSARILY FORMALLY REAL FIELDS

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ABSTRACT. The set of classical orderings of a field compatible with a given place from the field to the real numbers is known to be bijective with the set of homomorphisms from the value group of the place into the two element group. This fact is generalized here to the set of "generalized orders" compatible with an "extended absolute value," i.e., an absolute value allowed to take the value ∞. The set of extensions to a field F of a given generalized order on a subfield of F is computed and this computation is applied to count the number of such extensions that arise from finite degree field extensions of formally p-adic fields.

1. Introduction. The theory of formally p-adic fields had its origin in Ax and Kochen's "best possible" solution of a conjecture of Artin [**1, 13**]; its further development very much used the theory of formally real fields as a model and inspiration [**9, 12**]. That there are analogies between the two theories should not be surprising; after all, a field is formally real or formally p-adic if and only if it admits a place into the field of real numbers $\mathbf{R}$ or the field of p-adic numbers $\mathbf{Q}_p$, and $\mathbf{R}$ and the fields $\mathbf{Q}_p$ are simply the completions of the rational field $\mathbf{Q}$ at its nontrivial absolute values. These parallels suggest the possibility of a common theory which applies to fields admitting a place into a specific field of characteristic zero which, like $\mathbf{R}$ or $\mathbf{Q}_p$, comes equipped with a specific absolute value. Here is an example of a result in this direction. One of the major theorems of the Artin-Schreier theory of formally real fields is the fact that the set of orderings of a formally real field is naturally bijective with the set of real closures of the field; an equivalent version of this result says that the set of orderings P of a field compatible with a place τ into $\mathbf{R}$, i.e., with $\tau(P) \geq 0$, is naturally bijective with the set of real closures of the field admitting a place into

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