EXTENSIONS, DILATIONS AND FUNCTIONAL MODELS OF SINGULAR STURM-LIOUVILLE OPERATORS

BILENDER P. ALLAHVERDIEV

ABSTRACT. A space of boundary values is constructed for minimal symmetric singular Sturm-Liouville operator acting in the Hilbert space $L_w^2[a,b)$, $-\infty < a < b \leq \infty$, with deficiency indices $(2,2)$ (in Weyl's limit-circle case). A description of all maximal dissipative, maximal accretive, self-adjoint, and other extensions of such a symmetric operator is given in terms of boundary conditions at end points a and b. We investigate maximal dissipative operators with general (coupled or separated) boundary conditions. We construct a self-adjoint dilation of the maximal dissipative operator and its incoming and outgoing spectral representations, which makes it possible to determine the scattering matrix of the dilation. We also construct a functional model of the maximal dissipative operator and determine its characteristic function. We prove the theorem on completeness of the system of eigenfunctions and associated functions of the maximal dissipative operators.

1. Introduction. The theory of extensions of symmetric operators is one of the basic directions in operator theory. The first fundamental results in this theory were obtained by von Neumann [**17**], although the apparent origins can be found in the famous works of Weyl, see [**22**]. The theorems on representation of linear relations turned out to be useful for the description of various classes of extensions of symmetric operators. The first result of this type is due to Rofe-Beketov [**18**]. Kochubei [**11**] and Bruk [**3**] independently introduced the term 'space of boundary values' and in terms of this notion all maximal dissipative, maximal accretive, self-adjoint, and other extensions of symmetric operators, see [**9**] (also in the survey article [**8**]). However, regardless of the general scheme, the problem of the description of the maximal dissipative (accretive), self-adjoint and other extensions of a given symmetric operator via the boundary conditions is of considerable interest. This problem is particularly interesting in the case of singular