SOME NEW IYENGAR TYPE INEQUALITIES

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ABSTRACT. Some new Iyengar type inequalities for an integral are obtained by using the generalized Taylor formula with integral remainder.

1. Introduction. Let $f(x)$ be a differentiable function on a closed interval $[a, b]$ such that $\left| f'(x) \right| \le M$. Then

$$(1) \quad \left| \int_a^b f(x)\, \mathrm{d}x - \frac{(b-a)[f(a)+f(b)]}{2} \right| \le \frac{(b-a)^2 M}{4} - \frac{[f(b)-f(a)]^2}{4M}.$$

In 1938, Iyengar [16] established inequality (1) by using a geometric approach. So, we call (1) the Iyengar inequality.

Using the Rolle and Lagrange mean value theorems, the following inequalities were obtained naturally and simply in [27], producing a refinement of the Iyengar inequality (1).

Theorem A. *Let $f(x)$ be continuous on the closed interval $[a, b]$ and differentiable in the open interval (a, b), and let $m \le f'(x) \le M$ for*