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THE REPRESENTATIONS OF D^1

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ABSTRACT. In this paper we construct explicitly all irreducible representations of the norm one elements group in the quaternion division algebra over a local *p-field* where p is an *odd prime number*.

1. Introduction and notation. In this paper we will construct explicitly all irreducible representations of D^1, *the norm one elements group of* D, where D is the quaternion division algebra over a local p-field for an odd prime number p. Our motivation for finding representations of D^1, in addition to its own interest, is that they are needed to construct the representations of $U(2)$, the nonsplit unitary group in two variables, in relation to the reductive dual pair $(U(1), U(2))$ in the symplectic group $Sp(4)$. Some authors have studied the representations of division algebras in general [**1**]. Here we will be using the method used by Manderscheid [**10**] to construct the representations of $SL(2)$, to parametrize explicitly the representations of D^1. This method was briefly outlined, without details or proofs in [**11**]. We provide here the details and the proofs, getting the explicit inducing data in [**11**]. Although influenced by [**1**], this data does not follow from [**1**].

This paper consists of three sections. The first section is devoted to the basic results about the structure of D^1, its normal subgroups and their characters. In the second section we find all representations of D^1 whose dimensions are bigger than one. Finally in the last section after constructing all one-dimensional representations of D^1 we state and prove Theorem 3.5 which formalizes all the results obtained in Sections 2 and 3.

Let F be a non-Archimedean local p-field where p is an odd prime. Let $O = O_F$ be the ring of integers of F, and let ϖ be a generator of the maximal ideal $P = P_F$ in $O = O_F$. Let $k = k_F$ denote the residual class field O/P, and let q be the cardinality of k.

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