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REFLECTIONS ON SYMMETRIC POLYNOMIALS AND ARITHMETIC FUNCTIONS

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ABSTRACT. In an isomorphic copy of the ring of symmetric polynomials we study some families of polynomials which are indexed by rational weight vectors. These families include well known symmetric polynomials, such as the elementary, homogeneous, and power sum symmetric polynomials. We investigate properties of these families and focus on constructing their rational roots under a product induced by convolution. A direct application of the latter is to the description of the roots of certain multiplicative arithmetic functions (the core functions) under the convolution product.

Introduction. This paper is concerned with a certain isomorphic copy of the ring of symmetric polynomials, namely the ring of isobaric polynomials denoted by Λ'_k, where the isomorphism is given by a polynomial map involving the elementary symmetric polynomials. This ring is a polynomial ring with coefficients taken to be either the integers $\mathfrak{Z}$ or the rationals $\mathfrak{Q}$, and the image of a symmetric polynomial under the isomorphism mentioned above will be called an isobaric reflect. An isobaric[1] polynomial is one of the form $P_n = \sum_\alpha A(\alpha) t_1^{\alpha_1} \cdots t_k^{\alpha_k}$, where $\alpha = (\alpha_1, \dots, \alpha_k)$, $\alpha_i \geq 0$ are integers with $\sum_{j=1}^k j\alpha_j = n$.

As for the ring of symmetric polynomials, we can allow either a finite number k of variables or we can work in $\Lambda' = \oplus_{k\geq 0}\Lambda'_k$ with infinitely many variables. Families of isobaric polynomials occur in many contexts in mathematics. In [**5**] it was shown that the reflects of the complete symmetric polynomials (CSP) determine the multiplicative arithmetic functions locally. In [**6**] it was shown that the reflects of the power sum symmetric polynomials (PSP) determine the lattice of root fields of quadratic extensions. Properties of these two sequences of polynomials were discussed in [**7**] where the CSP-reflects are called Generalized Fibonacci Polynomials (GFP), and the PSP-reflects are called the Generalized Lucas Polynomials (GLP). Recall that the Complete

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