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COMPOSITION FOLLOWED BY DIFFERENTIATION BETWEEN BERGMAN AND HARDY SPACES

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ABSTRACT. Let Φ be an analytic self-map of the disc, and let H^p denote the Hardy space. The operator DC_Φ is defined for functions analytic in the disc by $DC_\Phi(f) = (f \circ \Phi)'$. We show that compactness and boundedness of the map $DC_\Phi : H^p \to H^q$, $p, q \geq 1$, are equivalent to the conditions $\Phi' \in H^q$ and $||\Phi||_\infty < 1$. For $\alpha > -1$ and $p \geq 1$, A^p_α denotes the weighted Bergman space. In the case $1 \leq p \leq q$, $DC_\Phi : A^p_\alpha \to A^q_\beta$ is bounded if and only if a related measure obeys a Carleson-type condition. Compactness is characterized by the analogous little-oh condition. For $1 \leq q < p$, Khinchine's inequality is used to show that boundedness and compactness are equivalent to an integrability condition on a weighted integral.

1. The Hardy space H^p, $p \geq 1$, is the Banach space of functions analytic in $U = \{z : |z| < 1\}$ satisfying

$$\|f\|_{H^p} = \sup_{0<r<1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(\mathrm{r}e^{i\theta})|^p \, d\theta \right\}^{1/p} < \infty.$$

References for the Hardy spaces include [**2**] and [**3**].

Let Φ be a nonconstant self-map of U, and let $C_\Phi(f) = f \circ \Phi$ for functions f analytic in the disc. Many authors [**1, 6, 7, 10**] have studied boundedness and compactness of C_Φ on the Hardy spaces. It is known [**12**] that if C_Φ is compact on H^p for some $p \geq 1$, then C_Φ is compact on all the Hardy spaces. Shapiro [**11**] characterized the self-maps Φ for which C_Φ is compact on H^2.

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