ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 3, 2005

THE PROJECTIVE AND INJECTIVE TENSOR PRODUCTS OF $L^p[0,1]$ AND X BEING GROTHENDIECK SPACES

QINGYING BU AND G. EMMANUELE

ABSTRACT. Let X be a Banach space and $1 < p$, $p' < \infty$ such that $1/p + 1/p' = 1$. Then $L^p[0,1]\hat{\otimes}X$, respectively $L^p[0,1]\check{\otimes}X$, the projective, respectively injective, tensor product of $L^p[0,1]$ and X, is a Grothendieck space if and only if X is a Grothendieck space and each continuous linear operator from $L^p[0,1]$, respectively $L^{p'}[0,1]$, to X^*, respectively X^{**}, is compact.

1. Introduction. In [**1, 4, 5**], Bu, Diestel, and Dowling gave a sequential representation of $L^p[0,1]\hat{\otimes}X$, the projective tensor product of $L^p[0,1]$ and X when $1 < p < \infty$. By this sequential representation, they showed that $L^p[0,1]\hat{\otimes}X$, $1 < p < \infty$, has the Radon-Nikodym property (respectively the analytic Radon-Nikodym property, the near Radon-Nikodym property, contains no copy of c_0) if and only if X has the same property. Using this sequential representation, Bu in [**2**] showed that $L^p[0,1]\hat{\otimes}X$, $1 < p < \infty$, contains no copy of l_1 if and only if X contains no copy of l_1 and each continuous linear operator from $L^p[0,1]$ to X^* is compact, and he also in [**3**] discussed all these geometric properties in $L^p[0,1]\check{\otimes}X$, the injective tensor product of $L^p[0,1]$ and X when $1 < p < \infty$.

In [**9**], Emmanuele showed that if X and Y are Grothendieck Banach spaces, one of which is reflexive, and if each continuous linear operator from X to Y^* is compact, then $X\hat{\otimes}Y$, the projective tensor product of X and Y, is a Grothendieck space. And he also in [**10**] showed that if $X\hat{\otimes}Y$ is a Grothendieck space and Y^* has the (b.c.a.p), then each continuous linear operator from X to Y^* is compact. As a special case of Emmanuele's results, we have that if X has the (b.c.a.p), then $L^p[0,1]\hat{\otimes}X$, $1 < p < \infty$, is a Grothendieck space if and only if X is a Grothendieck space and each continuous linear operator from $L^p[0,1]$

2000 AMS *Mathematics Subject Classification.* Primary 46M05, 46B28, 46E40.
Received by the editors on October 2, 2002, and in revised form on July 15, 2003.