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SELF-TRANSVERSAL SPACES AND THEIR DISCRETE SUBSPACES

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ABSTRACT. A space X is called *self-transversal* if there is a bijection $\varphi : X \to X$ such that the family $\tau(X) \cup \varphi(\tau(X))$ forms a subbase of the discrete topology on X. We prove that, under CH, there exists a compact scattered space which is not self-transversal. It is shown that there exist compact self-transversal spaces of arbitrarily large cardinality with the Souslin property. We present examples of compact spaces which give a negative answer in ZFC to Problems 2 and 3 from [**8**] and a partial negative answer to Problem 1 of [**8**]. We also establish that it is independent of ZFC whether any metrizable space X is self-transversal if and only if $w(X) = |X|$. We show that any monotonically normal scattered space is self-transversal and that adding a single point to a self-transversal space can destroy self-transversality.

1. Introduction. Recall that two topologies τ and μ on the same set X are called *transversal* if $\tau \cup \mu$ is a subbase for the discrete topology on X. A natural way of exploring the properties of a given space (X, τ) is to study the interaction of its topology with its copies on the same set obtained by all possible bijections. If some of these copies are transversal to τ then the space (X, τ) is called self-transversal.

The study of transversal topologies was initiated in 1966 by Steiner who proved in [**9**] that no countable infinite set X admits a pair of Hausdorff transversal topologies whose intersection is the cofinite topology on X (such topologies are called T_2-complementary). Later, intensive study of T_1-complementary topologies was undertaken by

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