ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 4, 2005

TOPOLOGICAL DESCRIPTION OF A NON-DIFFERENTIABLE BIOECONOMICS MODEL

E. GONZÁLEZ-OLIVARES, E. SÁEZ, E. STANGE, I. SZÁNTÓ

ABSTRACT. A predator-prey model with non-differentiable functional response and with a Cobb-Douglas type production function is considered. We show that the non-differentiability has a strong influence on the dynamics of the model, locally and globally. We prove that there is not a uniqueness of solutions for any initial conditions on the coordinate axis. We conclude that for any conditions of the parameters, the dynamics of the model does not contain a globally attracting singularity. Finally, in the parameters space, we prove the existence of an open set such that, for all values in this set, the model has at least two small amplitude limit cycles generated by Hopf bifurcations.

1. Introduction. Let us consider the family of vector fields $X_\mu^{\alpha,\beta}$ where

$$X_\mu^{\alpha,\beta} : \begin{cases} dx/dt = rx(1-(x/K)) - qx^\alpha y^\beta \\ dy/dt = b(pqx^\alpha y^\beta - cy) \end{cases} \tag{1}$$

This system describes the dynamics of an open access fishery model, where for each time $t > 0$, $x = x(t)$ is the size of the fishing resource and $y = y(t)$ the effort realized by the predator (man, industrial fisheries, etc.)

System (1) is defined on the region $\bar{\Omega} = \{(x, y) \in \mathbf{R}^2 \mid x \geq 0\ ,\ y \geq 0\}$, where $\mu = (r, K, p, q, b, c) \in \mathbf{R}_+^6$ and $0 < \alpha, \beta < 1$ denotes the bioeconomics parameters which have the following meanings:

2000 AMS *Mathematics Subject Classification.* Primary 92D25, 34C, 58F14, 58F21.
Key words and phrases. Stability, limit cycles, bifurcations, bioeconomic model.
The first author was financed partially by FONDECYT No. 1010399 and UCV No. 124778/2001.
The second and fourth authors were financed partially by USM Grant No. 120322 and FONDECYT No. 1030264.
The third author was financed partially by DIPUV Grant No. 09/2002.
Received by the editors on September 16, 2003, and in revised form on December 26, 2003.