ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 4, 2005

ON THE VANISHING OF THE ETA INVARIANT OF DIRAC OPERATORS ON LOCALLY SYMMETRIC MANIFOLDS

MARIA G. GARGOVA FUNG

ABSTRACT. In this note we prove a vanishing theorem for the Eta invariant of the spin Dirac operator on a locally symmetric space.

1. Introduction. Atiyah, Patodi and Singer [**2**] first defined the η-invariant of any self-adjoint elliptic operator A on a compact manifold as a measure of the asymmetry of $\mathrm{Spec}\,(A)$. If X is a compact oriented odd-dimensional locally symmetric manifold, then the generalized Dirac operator $\mathbf{D}$ (after choosing the essentially unique G-invariant connection) associated to a locally homogeneous Clifford module bundle over X is such an operator. Relying on Selberg trace formula analysis, Moscovici and Stanton [**7**] prove

Theorem 1.1. *Let G be a semi-simple Lie group with a maximal compact subgroup K, and let $\dim (G/K)$ be odd. Suppose that Γ is a cocompact discrete torsion free subgroup and suppose G has no factors locally isomorphic to $SL(3,\mathbf{R})$ or $SO(p,q)$, for p,q odd. Then for the generalized Dirac operator $\mathbf{D}$ on $\Gamma\backslash G/K$*

$$\eta(\mathbf{D}) = 0. \tag{1}$$

In this note we present another proof of this theorem which is not based on an evaluation of the trace of the odd heat kernel operator $\mathbf{D}e^{-t\mathbf{D}^2}$ by means of orbital integrals. Our proof is modeled after the proof of the vanishing theorems of cohomology of the locally symmetric space $\Gamma\backslash G/K$ and in particular after the algebraic proof of the triviality of the analytic torsion $\tau_1(\Gamma\backslash G/K)$ for the trivial representation of Γ in Speh [**8**]. In 3.1 we expand $Tr(\mathbf{D}e^{-t\mathbf{D}^2})$ using representation-theoretic data involving certain unitary representations of G. Then in

Received by the editors on November 18, 2002.