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ON THE JULIA SETS OF TWO PERMUTABLE ENTIRE FUNCTIONS

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ABSTRACT. In 1958, Baker posed a question that if f and g are two permutable transcendental entire functions, must their Julia sets be the same? Since then several classes of entire functions have been exhibited to support an affirmative answer to the question. In this paper, we shall complement or improve some of these results by exhibiting some new classes of entire functions.

1. Introduction and main results. Let f be a nonconstant entire function, and denote by f^n the nth iterate of f. The Fatou $F(f)$ set of f is the set of $z \in \mathbf{C}$ (the whole complex plane) where the family $\{f^n\}$ is normal in a neighborhood of z. Denote by $J(f)$ the complement of $F(f)$, which is called the Julia set of f. An obvious property of a Julia set for an entire or rational function f is that $J(f) = J(f^n)$. More of the basic results of the iteration of rational functions can be found in [**3, 4, 11, 19**] for transcendental entire or meromorphic functions. Factorization theory of entire or meromorphic functions is a subject which studies when an entire or meromorphic function F can be expressed as the composition of two or more simpler entire or meromorphic functions. Here a function h is said to be simpler than another function k means that h has a growth much slower than that of k's. If F can be expressed as $F = f \circ g$, then f and g are called left and right factors of F, respectively. For more of the details, developments and related results of the factorization theory, we refer the reader to [**5, 8**]. As iteration is a special case of the composition, from this it is easily understood that factorization theory and complex dynamics are closely related to each other. An entire or meromorphic function is called prime (pseudo-prime) f if, whenever $f = g \circ h$ for some meromorphic functions g and h, then either g or h is linear (g rational

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