ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 5, 2005

QUADRIC, CUBIC AND QUARTIC CONES

ANATOLY B. KORCHAGIN AND DAVID A. WEINBERG

ABSTRACT. There are 2 irreducible quadric cones (real and imaginary) required for obtaining the affine classification of the 4 irreducible conic sections. According to Newton there are 5 irreducible cubic cones required for obtaining his classification of 59 irreducible cubic sections. In this historical survey paper we show how it follows from Gudkov's classification of forms of real projective quartic curves that 1037 quartic cones are required for obtaining a similar classification of irreducible quartic sections. We also present the singular-isotopy classification of the unions of irreducible affine cubic curves with their asymptotes, which consists of 99 classes. This classification sheds a new light on Newton's famous classification consisting of 78 species.

1. Conic sections. Menaechmus (ca. 350 B.C.) was the first to describe a classification of real nonempty irreducible conic sections. His approach was geometrical. He considered three kinds of cones: acute, right and obtuse cones. For each of the cones, he drew a plane perpendicular to a generator of the cone through a point of the generator other than the vertex. For the acute, right and obtuse cones, he obtained an ellipse, parabola and hyperbola, respectively. He considered cones with one nappe, which are generated by a semi-line, and thus his hyperbola only had one branch. If one considers a cone generated by a line, then the cone of two nappes is obtained and the second branch of the hyperbola appears. Usually one obtains ellipse, parabola and hyperbola by intersecting one cone with various planes.

The next advance in the study of conic sections is connected with the algebraization of the subject whereby a conic section is described by an equation. Such an algebraization can be found in *Conics* by Apollonius. For example, in the case of the parabola he proved that for any point M lying on the parabola with vertex P the equation

2000 AMS *Mathematics Subject Classification.* Primary 01A99, 01A45, 01A60, Secondary 14R05, 14A25, 14H50, 51N35, 57M99, 57N99.
Key words and phrases. Topological and isotopy classifications of real algebraic curves.
Received by the editors on May 1, 2002, and in revised form on September 12, 2002.