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ON THE GAUSS MAP OF RULED SURFACES IN MINKOWSKI SPACE

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ABSTRACT. In this paper, we study some characterization of ruled surfaces in Minkowski space in terms of the Gauss map. We give new examples of cylindrical and noncylindrical ruled surfaces in a 4-dimensional Minkowski space with the pointwise 1-type Gauss map.

1. Introduction. Since the late 1970's when B.-Y. Chen introduced the theory of finite type immersion, its study has been extended to the submanifolds of pseudo-Euclidean spaces, namely a pseudo-Riemannian submanifold M of an m-dimensional pseudo-Euclidean space $\mathbf{E}_s^m$ with signature $(s, m-s)$ is said to be of *finite type* if its position vector field x can be expressed as a finite sum of eigenvectors of the Laplacian Δ of M, that is,

$$x = x_0 + x_1 + x_2 + \cdots + x_k,$$

where x_0 is a constant map, $x_1, \ldots, x_k$ nonconstant maps such that $\Delta x_i = \lambda_i x_i, \lambda_i \in \mathbf{R}$, $i = 1, 2, \ldots, k$, [**3, 7**]. If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are different, then M is said to be of *k-type*. Similarly, we can apply this notion to a smooth map, for example, the Gauss map G that is one of the most natural smooth maps on an n-dimensional pseudo-Riemannian submanifold M of $\mathbf{E}_s^m$. Thus, the Gauss map G is said to be of *finite type* if G is a finite sum of $\mathbf{E}_s^m$- valued eigenfunctions of Δ [**2, 4**]. We also similarly define the notion of k-type Gauss map on M as usual.

There are many examples of submanifolds in the Minkowski space $\mathbf{E}_1^m$ with finite type Gauss map, for example, B-scrolls in $\mathbf{E}_1^3$, several kinds of cylinders and extended B-scrolls in $\mathbf{E}_1^4$ are those with 1-type

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