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WARING'S PROBLEM FOR LINEAR POLYNOMIALS AND LAURENT POLYNOMIALS

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ABSTRACT. Waring's problem is about representing any function in a class of functions as a sum of kth powers of nonconstant functions in the same class. We allow complex coefficients in these kind of problems. Consider $\sum_{i=1}^{p_1} f_i(z)^k = z$ and $\sum_{i=1}^{p_2} f_i(z)^k = 1$. Suppose that $k \geq 2$. Let p_1 and p_2 be the smallest numbers of functions that give the above identities. W.K. Hayman obtained lower bounds of p_1 and p_2 for polynomials, entire functions, rational functions and meromorphic functions. First, we consider Waring's problem for linear polynomials and get $p_1 = k$ and $p_2 = k+1$. Then, we study Waring's problem for Laurent polynomials and obtain lower bounds of p_1 and p_2.

1. Introduction.

1.1. Waring's problem. Waring's problem deals with representing any function in a class of functions as a sum of kth powers of nonconstant functions in the same class. We allow complex coefficients in these problems.

Let k and n be natural numbers. Consider the equation of the form

$$\sum_{i=1}^{n} f_i(z)^k = Q(z), \tag{1.1.1}$$

where $f_1, f_2, \dots, f_n$ and Q are nonconstant polynomials with complex coefficients. Suppose that

$$f_1(z)^k + f_2(z)^k + \cdots + f_n(z)^k = z. \tag{1.1.2}$$

Then we obtain

$$f_1(Q(z))^k + f_2(Q(z))^k + \cdots + f_n(Q(z))^k = Q(z) \tag{1.1.3}$$

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