HYPERBOLIC TWISTOR SPACES

D.E. BLAIR, J. DAVIDOV AND O. MUŠKAROV

Dedicated to Professor Paulette Libermann

ABSTRACT. In contrast to the classical twistor spaces whose fibres are 2-spheres, we introduce twistor spaces whose fibres are hyperbolic planes over manifolds with (almost) paraquaternionic structures; the definition of these structures is based on the notion of an almost quaternionic structure of the second kind in the sense of P. Libermann. We discuss two natural almost complex structures on such a twistor space and their holomorphic functions.

1. Introduction. In this paper we introduce hyperbolic twistor spaces which are bundles over manifolds with almost quaternionic structure of the second kind in the sense of P. Libermann and whose fibres are hyperbolic planes. These spaces admit two natural almost complex structures defined as in the classical twistor space theory. Our main purpose is to study their differential-geometric properties as well as the existence of holomorphic functions (part of the results have been announced in the first author's lecture [**4**]). In Section 2 we define hyperbolic twistor spaces and their almost complex structures when the base manifolds are of dimension ≥ 8, developing the theory for paraquaternionic Kähler manifolds. In Section 3 we give the corresponding treatment for base manifolds which are four-dimensional with a metric of signature $(++--)$. Finally in Section 4 we treat the question of existence of holomorphic functions on hyperbolic twistor spaces and show that, in contrast to the classical case, there can be an abundance of (global) holomorphic functions on a hyperbolic twistor space.