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HYPERSURFACES SINGULAR ALONG SMOOTH NONLINEARLY NORMAL CURVES

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ABSTRACT. Let $X \subset \mathbf{P}^n$ be a smooth curve and $X^{(1)}$ the first infinitesimal neighborhood of X in $\mathbf{P}^n$. Here we prove that $X^{(1)}$ has maximal rank for several nonlinearly normal embeddings $X \subset \mathbf{P}^n$.

1. Introduction. Let $X \subset \mathbf{P}^n$ be a smooth curve and $X^{(1)}$ the first infinitesimal neighborhood of X in $\mathbf{P}^n$, i.e., the closed subscheme of $\mathbf{P}^n$ with $(\mathbf{I}_X)^2$ as the ideal sheaf. Thus $X^{(1)}_{\mathrm{red}} = X$. A hypersurface Z of $\mathbf{P}^n$ is singular along X if and only if it contains $X^{(1)}$. Thus the Hilbert function of $X^{(1)}$, i.e., the string of integers $h^0(\mathbf{P}^n, \mathbf{I}_{X^{(1)}}(t))$, $t \geq 0$, is a natural numerical invariant of X. A few papers were devoted to the computation of the Hilbert function of $X^{(1)}$ when X is either a canonically embedded curve or a linearly normal curve of genus g and large degree, say degree $d \geq 2g+3$, [**5–8**]. Here we will consider the case in which C is not linearly normal. Here are our results.

Theorem 1.1. *Fix integers n, d and g, and set $x := d+1-g-n$. Assume $x \geq 2$, $n \geq x+5$, $d-x-1 \geq 2g+3$, $g \leq n-x-2$ and $(n-x)(n-x-1)/2 \geq 2(d-x-2)+1-g$. Let X be a smooth connected projective curve of genus g and $L \in \mathrm{Pic}^d(X)$. Then there is an embedding $j : X \to \mathbf{P}^n$ such that $j^*(\mathbf{O}_{j(X)}(1)) \cong L$ and $h^1(\mathbf{P}^n, \mathbf{I}_{j(X)^{(1)}}(k)) = 0$ for every $k \geq 3$. Furthermore, $h^0(\mathbf{P}^n, \mathbf{I}_{j(X)^{(1)}}(2)) = 0$ and $j(X)^{(1)}$ has maximal rank.*

For instance, if $X \subset \mathbf{P}^n$ is a genus two smooth curve of degree 25, then Theorem 1.1 covers the cases $17 \leq n \leq 22$.

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