ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 6, 2005

EXACT NUMBER OF SOLUTIONS FOR SINGULAR DIRICHLET BOUNDARY VALUE PROBLEMS

ZHONGLI WEI

ABSTRACT. This paper establishes the exact multiplicities and properties of positive solutions for singular Dirichlet boundary value problems of second order ordinary differential equations.

1. Introduction. In this paper we study the exact multiplicities and properties of positive solutions of the following singular boundary value problems

$$(1_\lambda) \qquad \begin{cases} -x''(t) = \lambda(x^q(t) + kx(t) + x^{-m}(t)) & t \in (0,1), \\ x(t) > 0 & t \in (0,1), \\ x(0) = x(1) = 0 \end{cases}$$

where λ is a parameter, $k \geq 0$ is a constant and $q,\ m$ satisfy either

(H1) $0 \leq m \leq 1/3$, $1 < q < \infty$; or

(H2) $1/3 < m < 1$, and

$$1 < q < 1 + \left[\frac{1+m}{2(3m-1)}\right]\left[(3-5m) + \sqrt{(3-5m)^2 + 8(3m-1)(1-m)}\right].$$

By singularity we mean that the function $f = \lambda(x^{-m}(t) + x^q(t) + kx(t))$ in (1_λ) is unbounded at the end points $t = 0$ and $t = 1$. A function $x(t) \in C[0,1] \cap C^2(0,1)$ is called a $C[0,1]$ positive solution of (1_λ) if it satisfies (1_λ). A $C[0,1]$ positive solution of (1_λ) is called a $C^1[0,1]$ positive solution if $x'(0^+)$ and $x'(1^-)$ both exist.

Problem (1_λ) comes from a problem raised by Agarwal and O'Regan [**1**]. Agarwal and O'Regan proved the equation

$$\begin{cases} y''(t) + \delta(y^{-\alpha}(t) + y^{\beta}(t) + 1) = 0 & 0 < t < 1 \\ y(0) = y(1) = 0 & \delta > 0 \text{ a parameter} \end{cases}$$

2000 AMS *Mathematics Subject Classification.* Primary 34B15.
Key words and phrases. Singular boundary value problem, positive solution, exact structure of all solutions, properties of solutions.
Received by the editors on January 15, 2003, and in revised form on May 5, 2003.