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THE ASYMPTOTIC GROWTH OF EQUIVARIANT SECTIONS OF POSITIVE AND BIG LINE BUNDLES

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1. Introduction. Let X be a complex manifold, say of dimension n, and L an holomorphic line bundle on it. Suppose that a finite group G acts holomorphically on X and that this action lifts to a holomorphic action on L, so that for every $x \in X$ and $g \in G$ the induced map $L_x \to L_{gx}$ is complex linear. Then G also acts on every tensor power $L^{\otimes k}$, and thus it acts linearly on the spaces $H^0(X, L^{\otimes k})$ of global holomorphic sections of $L^{\otimes k}$. Therefore, $H^0(X, L^{\otimes k})$ splits G-equivariantly in terms of the irreducible representations of G. Let $H^0(X, L^{\otimes k})_i$ be the equivariant summand corresponding to the ith irreducible representation. The first object of this paper is the asymptotic growth of the dimension of $H^0(X, L^{\otimes k})_i$ as $k \to +\infty$, in the following two situations: i) X is complex projective and L is ample, and ii) X is complex projective and L is *big*, that is, it has maximal Kodaira dimension. We shall then discuss a generalization of this to the symplectic category.

In spite of its very short and simple proof, perhaps the main result of this article is Theorem 3, which deals with case i). This case has been studied (in a broader algebraic formulation and with algebraic techniques) by various authors, see [**13, 6, 7**]; the two latter papers deal with the actions of reductive groups. It follows from this body of work that in our complex projective, ample situation $\dim H^0(X, L^{\otimes k})_i = a_i k^n + b_i k^{n-1} + O(k^{n-2})$, where a_i and b_i are described algebraically. Here we give, in terms of the Riemann-Roch polynomial of L, an asymptotic estimate which is in many cases more refined, if the dimension d of the locus of points in X with nontrivial stabilizer in G is taken into account. More precisely, we give an explicit asymptotic expansion with a remainder which is $o(k^{d+1})$; this is thus strictly more informative (of course, in our complex geometric situation) if $d \leq n-3$, and equally fine if $d = n-2$.

Our new approach is analytic and based on the study of the Szegő kernel Π of L; the key ingredient is the off-diagonal estimate discussed

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