

## WEIGHTED COMPOSITION OPERATORS ON NON-LOCALLY CONVEX WEIGHTED SPACES

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**ABSTRACT.** Let  $(A, \tau)$  be a topological vector space,  $X$  and  $Y$  Hausdorff completely regular spaces and  $V$  and  $U$  Nachbin families on  $X$  and  $Y$  respectively. For a pair of maps  $\varphi : Y \rightarrow X$  and  $\psi : Y \rightarrow \mathcal{L}(A)$ ,  $\mathcal{L}(A)$  being the vector space of continuous operators from  $A$  into itself, we study the conditions under which the corresponding weighted composition operator  $\psi C_\varphi$ , assigning to each  $f \in CV(X, A)$  the function  $y \mapsto \psi_y(f \circ \varphi(y))$ , maps a subspace  $E$  of  $CV(X, A)$  continuously into another given subspace  $F$  of  $CU(Y, A)$ . We also examine when  $\psi C_\varphi$  is bounded, (locally) equicontinuous or (locally) precompact from  $E$  into  $F$ .

**1. Introduction.** The weighted composition operators  $uC_\varphi : f \mapsto uf \circ \varphi$  on the Banach algebra  $C(K)$  of scalar-valued continuous functions on a compact space  $K$  were studied by Kamowitz in [8]; where  $u \in C(K)$  and  $\varphi : K \rightarrow K$  is a continuous self map on  $K$ . Since then, numerous papers were published in connection with the subject in the scalar case and in the vector-valued one [6, 7, 10, 12, 15, 21, 22], etc. In the scalar case, Singh and Summers [21] studied the composition operators  $C_\varphi$  on the Nachbin weighted spaces  $CV(X)$  and  $CV_0(X)$ ,  $X$  being a Hausdorff completely regular space and  $V$  a Nachbin family on  $X$ . The so-called extended composition operators between weighted spaces were the subject of [14].

Jeang and Wong [7] dealt with the weighted composition operators  $uC_\varphi : f \mapsto uf \circ \varphi$  from  $C_0(X)$  into  $C_0(Y)$ , where  $X$  and  $Y$  are Hausdorff locally compact spaces,  $u \in C(Y)$  and  $\varphi$  a map from  $Y$  into  $X$ . For special function spaces, namely the Banach spaces of analytic functions on the unit disk, the multiplication operators were the subject of [4].

In the vector-valued setting, Jamison and Rajagopalan [6] considered the weighted composition operators  $\psi C_\varphi$  on the Banach space  $C(K, A)$ ,

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