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A NEW PROOF OF LIEBMANN CLASSICAL RIGIDITY THEOREM FOR SURFACES IN SPACE FORMS

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ABSTRACT. In this paper we provide a new direct proof of the Liebmann classical rigidity theorem for surfaces in space forms, showing that the only compact surfaces with constant Gaussian curvature which are immersed into the Euclidean space $\mathbf{E}^3$, into the hyperbolic space $\mathbf{H}^3$, or into an open hemisphere $\mathbf{S}^3_+$ are the totally umbilical round spheres. Our proof is an application of the Gauss-Bonnet theorem along with a formula involving the Gaussian curvatures of the first and second fundamental forms of the surface, which is interesting *per se*.

1. Introduction. In 1897 Hadamard [**6**] proved that an ovaloid, that is, a compact connected surface with positive Gaussian curvature, in the three-dimensional Euclidean space $\mathbf{E}^3$ is a topological sphere. In view of this result, it was natural to look for conditions which allowed one to conclude that such a surface was necessarily a totally umbilical round sphere. In 1899 Liebmann [**11**] obtained his celebrated rigidity result, which states that every compact connected surface in $\mathbf{E}^3$ with constant Gaussian curvature is necessarily a totally umbilical round sphere.

The most famous proof of Liebmann theorem was given by Hilbert, just a short time after Liebmann's original proof, using local computations [**7**, Appendix 5], see also [**4**], as well as [**3**, Theorem 1, p. 317] for

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