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THE DIVERGENCE-FREE JACOBIAN CONJECTURE IN DIMENSION TWO

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ABSTRACT. A special case, called the divergence-free case, of the Jacobian conjecture in dimension two is proved.

1. Introduction. This note outlines an argument for a special case of the Jacobian conjecture in dimension two: Suppose $F : C^2 \to C^2$ is a polynomial so that

$$(1) \qquad F(0) = 0, \quad F'(0) = I, \quad \det(F'(z)) = 1, \quad z \in C^2.$$

where I is the identity transformation on C^2.

Write

$$F(x,y) = \begin{pmatrix} r(x,y) + x \\ s(x,y) + y \end{pmatrix}, \quad (x,y) \in C^2$$

where r, s have no nonzero constant or linear terms and observe that

$$\det F' = \{r, s\} + \nabla \cdot \begin{pmatrix} r \\ s \end{pmatrix} + 1$$

so that (1) gives

$$(2) \qquad \{r, s\} + \nabla \cdot \begin{pmatrix} r \\ s \end{pmatrix} = 0$$

with

$$\{r, s\} = r_1 s_2 - r_2 s_1, \quad \nabla \cdot \begin{pmatrix} r \\ s \end{pmatrix} = r_1 + s_2,$$

the Poisson bracket and divergence respectively of the vector field (r, s), subscripts in these instances indicating partial derivatives in first and second arguments.

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