ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 1, 2006

REPRESENTATIONS AND INTERPOLATIONS OF WEIGHTED HARMONIC BERGMAN FUNCTIONS

KYESOOK NAM

ABSTRACT. On the setting of the upper half-space of the Euclidean n-space, we study representation theorems and interpolation theorems for weighted harmonic Bergman functions. Also, we consider the harmonic (little) Bloch spaces as limiting spaces.

1. Introduction. Let $\mathbf{H}$ denote the upper half space $\mathbf{R}^{n-1} \times \mathbf{R}_+$ where $\mathbf{R}_+$ denotes the set of all positive real numbers. We will write points $z \in \mathbf{H}$ as $z = (z', z_n)$ where $z' \in \mathbf{R}^{n-1}$ and $z_n > 0$.

For $\alpha > -1$ and $1 \le p < \infty$, let $b^p_\alpha = b^p_\alpha(\mathbf{H})$ denote the *weighted harmonic Bergman space* consisting of all real-valued harmonic functions u on $\mathbf{H}$ such that

$$\|u\|_{L^p_\alpha} := \left(\int_{\mathbf{H}} |u(z)|^p \, dV_\alpha(z) \right)^{1/p} < \infty$$

where $dV_\alpha(z) = z_n^\alpha dz$ and dz is the Lebesque measure on $\mathbf{R}^n$. Then we can see easily that the space b^p_α is a Banach space. In particular, b^2_α is a Hilbert space. Hence, there is a unique Hilbert space orthogonal projection Π_α of L^2_α onto b^2_α which is called the weighted harmonic Bergman projection. It is known that this weighted harmonic Bergman projection can be realized as an integral operator against the weighted harmonic Bergman kernel $R_\alpha(z, w)$. See Section 2.

In [**6**], many fundamental weighted harmonic Bergman space properties have been studied. In this paper, we study the representation property of b^p_α-functions and the interpolation by b^p_α-functions. Our methods are taken from those in [**4**] and based on estimates of the

2000 AMS *Mathematics Subject Classification.* Primary 31B05, 31B10, 30D45, 30D55.

This research was supported by the Korean Research Foundation grant KRF-2000-DP0014.
Received by the editors on June 18, 2003, and in revised form on October 6, 2003.