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A TELESCOPING PRINCIPLE FOR OSCILLATION OF SECOND ORDER DIFFERENTIAL EQUATIONS ON A TIME SCALE

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ABSTRACT. We establish a telescoping principle for oscillation of the second order scalar differential equation $(p(t)x^{\Delta}(t))^{\Delta} + q(t)x(\sigma(t)) = 0$ on a time scale and use it to obtain several new conditions for oscillation. Our results extend, unify, and modify the telescoping principles for oscillation of second order differential equations and difference equations, respectively, and provide a new tool for the investigation of oscillation on time scales. We illustrate the results obtained by several examples, none of which may be handled by known oscillation criteria.

1. Introduction. In this paper, we study the self-adjoint second order scalar equation

$$(p(t)x^{\Delta}(t))^{\Delta} + q(t)x(\sigma(t)) = 0 \tag{E}$$

on a time scale $\mathbf{T}$, that is, on a nonempty closed subset $\mathbf{T}$ of $\mathbf{R}$, the set of real numbers. Without loss of generality we assume throughout that $0 \in \mathbf{T}$ and $\sup \mathbf{T} = \infty$ since we are interested in extending oscillation and nonoscillation criteria for the corresponding differential and difference equations, namely

$$(p(t)x'(t))' + q(t)x(t) = 0 \tag{1.1}$$

with $\mathbf{T}$ the interval $[0, \infty)$, and

$$\Delta(p_n \Delta x_n) + q_n x_{n+1} = 0 \tag{1.2}$$

with $\mathbf{T} = \mathbf{N}_0$, the set of nonnegative integers.

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