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EXISTENCE OF SOLUTIONS FOR THE BAROTROPIC-VORTICITY EQUATION IN AN UNBOUNDED DOMAIN

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ABSTRACT. In this paper we consider the two-dimensional barotropic-vorticity equation in the first quadrant, and using a rearrangement variational principle, prove it has a solution. The solution represents a steady localized topographic ideal flow. The data given are the behavior of the flow at infinity, the rearrangement class of the vorticity field and the height of the localized seamount.

1. Introduction. In this paper we prove existence of solutions for the following barotropic-vorticity equation

$$[\psi, \omega + h] = 0, \tag{1.1}$$

satisfying

$$-\Delta\psi \in \mathcal{F} + h, \tag{1.2}$$

where $[\cdot,\cdot]$ denotes the Jacobian. Here $\mathcal{F}$ denotes a class of rearrangements of a given function, and h is some fixed non-negative function, see the next section for precise definitions. Equation (1.1) is the governing equation describing the flow of an ideal fluid with ψ representing the stream function, ω the vorticity and h the height of the bottom topography. In the present work we are assuming (1.1) to hold in a *non-symmetric* planar domain, the first quadrant Π_+. Since the domain is unbounded, we require some asymptotic condition to be satisfied; namely, we assume $\psi \to \lambda x_1 x_2$ at infinity ($\lambda x_1 x_2$ represents the stream function of an irrotational flow). We also assume that ω belongs to the class of rearrangements of a given function. Similar problems in *symmetric* domains have been considered but the methods are not applicable in our situation. We derive new estimates in order to overcome this lack of symmetry.

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